Porosity versus Depth Relationship Derived from Rock Mechanical Arguments

E. Fjær[1)], A.E. Lothe [1)], Ø. Sylta[2)]
[1)]SINTEF Petroleum Research, Trondheim, Norway, [2)] Migris, Trondheim, Norway

During deposition of sedimentary rocks, the porosity is gradually reduced as a result of the increasing weight of the overburden. Prior to diagenetic processes, the porosity at a given depth depends on the rock composition – which is defined during sedimentation, as well as stress, pore pressure and rock mechanical properties – which change gradually during the burial processes.

The aim with this work is to establish a relation between porosity versus depth based on simplified relation about stress path, rock compressibility, grain size and sea-floor porosity and relation between mean grain size and clay content.

Porosity versus depth – introduction

The porosity is largely dependent on the lithology (Figure 1). Thus, the clay amount become a critical factor when the porosity of sandstone and clay-rich sandstones should be predicted.

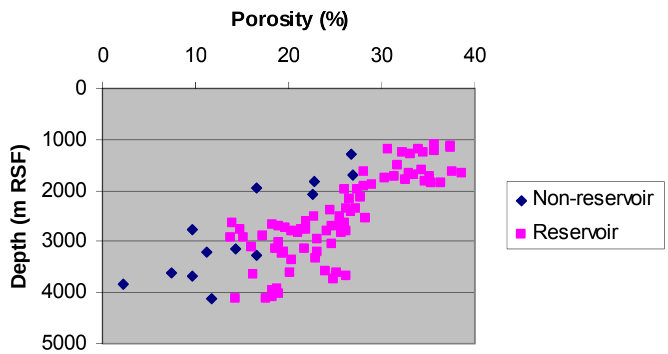

Figure 1: Measured porosity (%) versus depth from Haltenbanken, offshore Norway. Non-reservoir is clayrich sandstone or silt stone. Reservoir is sandstone units. Data from Ramm & Bjørlykke (1994).

Porosity versus Vshale

The porosity at $z = z_w$ (seafloor) may be estimated on the basis of Shumway (1960) as shown in the Figure 2. The solid line is a least square fit to the data, given as:

$$\phi_o = \phi_{cr} - \frac{1}{m}\ln\frac{d_{50}}{d_o} \qquad (1)$$

where ϕ_{cr} = 0.287, m = 11.2, d_{50} is the mean sediment diameter, and the reference diameter d_o = 1 mm. Notice that this formula gives obvious unphysical results if $d_{50} > 25\ d_o$. The range of validity of the formula is strictly speaking restricted to $0.001\ d_o < d_{50} < 0.5\ d_o$.

There is obviously a connection between clay content and mean sediment diameter. Assuming that this connection is given by a geometric mean of two discrete particle sizes, we find that

$$d_{50} = d_{ss}^{1-V_{sh}} d_{sh}^{V_{sh}} \qquad (2)$$

where d_{ss} is diameter sandstone grains, d_{sh} is diameter clay grains, V_{sh} is precent clay. Assuming further that d_{ss} = 400 μm and d_{sh} = 1 μm, we find that Eq. 1 can be written as

$$\phi_o = 0.369 + 0.535 V_{sh} \qquad (3)$$

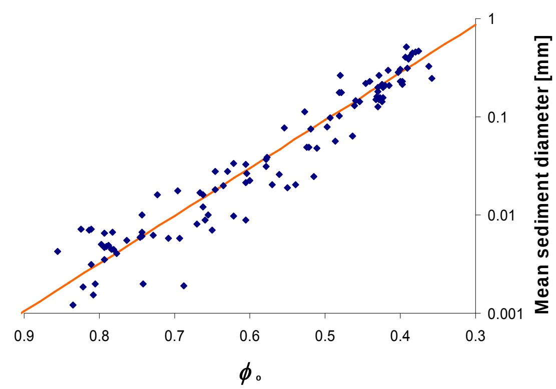

Figure 2 Depositional porosity with low effective stresses (Φ_0) versus mean sediment diameter (mm). The samples were collected 10 cm beneath the sea floor. Data from Shumway (1960).

SINTEF

Method

Porosity changes during compaction

$$d\phi \propto -d\sigma_z' \qquad (4)$$

where σ_z' is the effective vertical stress.

Since $d\Phi$=0 when Φ=0, and since it will be more easy to compact a rock with high porosity compared to a rock with low porosity, we than may assume that

$$d\phi \propto \phi \qquad (5)$$

From a simple rock mechanical consideration, we can deduce that

$$d\phi = -\frac{1}{3}\left(\frac{1-\phi}{K_{fr}} - \frac{1}{K_s}\right)\frac{1+\nu_{fr}}{1-\nu_{fr}}\, d\sigma_z' \qquad (6)$$

where Φ is the porosity, K_{fr} is the frame bulk modulus and K_s is the bulk modulus.

We assume the following relations for the frame moduli:

$$K_{fr} = K_s \frac{1-\phi}{1+\beta K_s \phi} \qquad (7)$$

$$\nu_{fr} = \nu_o + \nu_1 \phi \qquad (8)$$

The relations are shown in Figure 3.

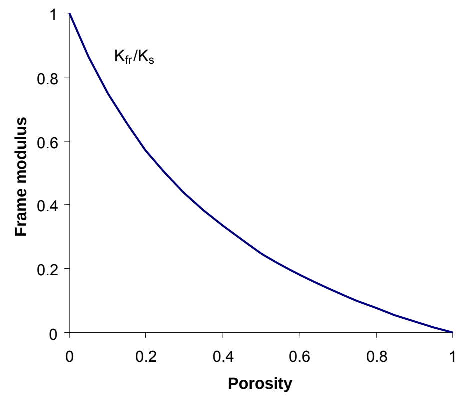

Figure 3: Frame moduli versus porosity, as given by Eq. 7 . The parameter product βK_s = 2.

Equation for the porosity

Combining Eqs. (6), (7) and (8) we find a differential equation for the porosity:

$$\frac{d\phi}{\phi(1-s\phi)} = -b\, d\sigma_Z' \qquad (9)$$

where

$$b = \frac{1}{3}\beta\frac{1+\nu_o}{1-\nu_o} \qquad (10)$$

$$s = -\frac{2\nu_1}{1-\nu_o^2} \qquad (11)$$

The solution of the equation is

$$\phi = \frac{\phi_o}{s\phi_o + (1-s\phi_o)e^{b\sigma_z'}} \qquad (12)$$

where Φ_0 is the porosity at sea-floor.

However, one can also make other assumtions for K_{fr} and ν_{fr} and get the same equation (Eq. 9) for the porosity (see also Figure 4).

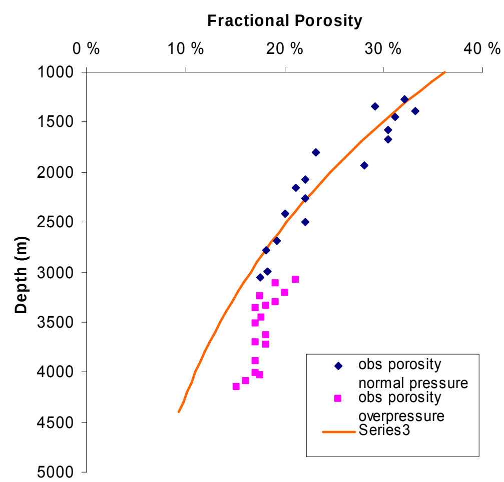

Figure 4 We find the best fit with shale data from Plumley (1980) if Φ_0=0.52, r=0.35 and q=0.428 km^{-1}. Waterdepth is set to zero.

SINTEF

The pore pressure changes with depth (z) as

$$\Delta p_f = g\rho_f \Delta z \qquad (13)$$

The vertical stress changes with depth as

$$\Delta\sigma_z = g\rho(z)\Delta z = g\left(\rho_s - \left(\rho_s - \rho_f\right)\phi\right)\Delta z \qquad (14)$$

Combined with eq. (5) we find

$$\sigma_z^{'} = \int_{z_w}^{z} g\left(\rho_s - \rho_f\right)(1-\phi)dz \qquad (15)$$

where z_w is the water depth. We now introduce the gradient G_z, defined as

$$G_z = \frac{1}{z - z_w}\int_{z_w}^{z} g\left(\rho_s - \rho_f\right)(1-\phi)dz \qquad (16)$$

For layer i, the porosity may then be written:

$$\phi(z) = \frac{\phi_{i,o}}{s_i\phi_{i,o} + \left(1 - s_i\phi_{i,o}\right)e^{b_i G_z (z - z_w)}} \qquad (17)$$

Note that s_i, b_i, and $\Phi_{i,0}$ are local parameters – specific for the actual layer under consideration.

In the result part, we have set $s=r$ and $b_iG_z=q$

Porosity in overpressured zones

If there is an overpressure Δp caused by incomplete drainage in one zone, the porosity will be lower than predicted in Eq. 17. We can estimate this by noticing that the porosity is a function of the effective vertical stress, so that the porosity at depth z will in this case be equal to the porosity at another depth z', defined by

$$\sigma_z(z') - g\rho_f z' = \sigma_z(z) - g\rho_f z - \Delta p \qquad (18)$$

An approximate solution will be

$$\phi' = \frac{\phi_o}{r\phi_o + \left(1 - r\phi_o\right)e^{q(z' - z_w)}} \qquad (19)$$

that gives the porosity in the overpressured zone.

Results

We wanted to test our formula Ex. 19 on different datasets. First we tested it on data from Yang & Aplin (2004). They have published log data from three different areas; two logs from the North Sea, one from Gulf of Mexico (GoM) and one log from West Africa. Yang & Aplin estimated several parameters, including porosity, pore pressure and clay content making use of neural network methods.

Figure 5 shows our estimated porosity versus assumed porosity (from Yang & Aplin) in a North Sea well. The clay content and overpressure are used as input in the formula. Only two free rock mechanical parameters are varied e.g. r=-1.2 and q=4.17*10^{-3}.

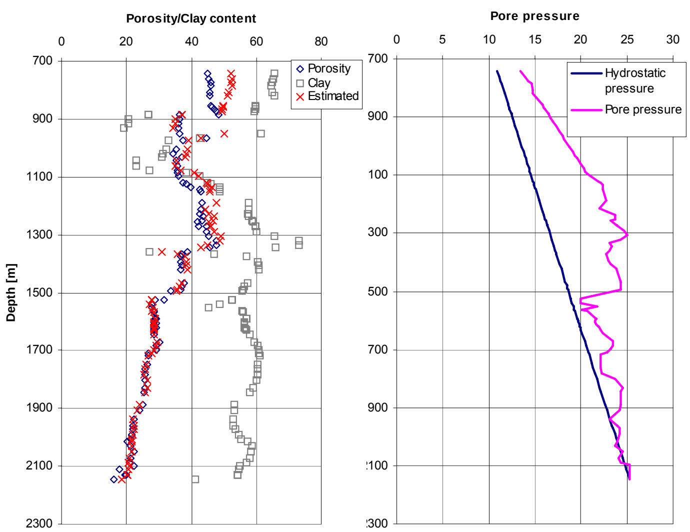

Figure 5: Observed porosity and clay content in a North Sea well versus depth. Our estimated porosity is shown in red. Reworked from Yang & Aplin (2004).

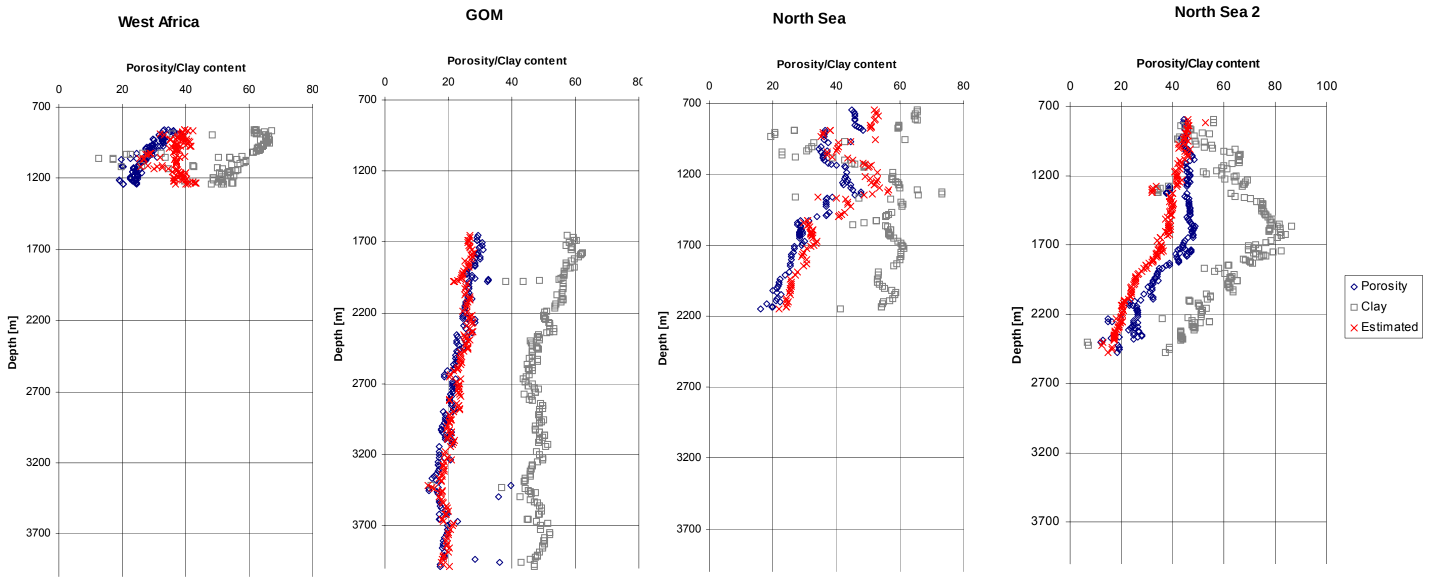

Figure 6: The porosity-depth relation is tested on published data from Yang & Aplin. Red graphs show our estimated porosity versus measured porosity.

The same two parameters are used to fit the data from all four wells: r=-1.2 and q=4.17*10^{-3}. A good match in three of the logs is seen between the estimated and the measured porosity (wells GOM and the North Sea wells; Figure 6). In the West Africa well our estimates are slightly higher that the measured porosity.

The overall results indicate that the parameter relationships generated by the neural network to a large extent agree with the theoretically based relationships generated here.

North Sea Data set

We also wanted to test the formula on data from the North Sea.

Figure 7 shows a comparison between observed and estimated porosity. Also the Vshale is shown. The input parameter is set to r=-5 and q=3.56*10^{-3}.

We get a good fit the well except between 2770 m and 2840 m depth. In this area, a higher porosity is observed correspondly with low shale content. If we estimate the pore pressure, a marked overpressure regime is estimated in this sandy unit. This is in line with measured overpressure from the well.

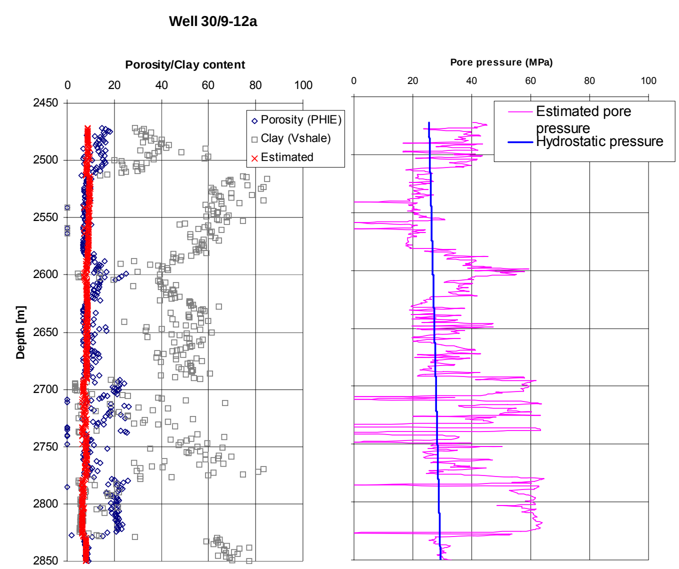

Figure 7: Left figure: Observed porosity and clay content in a North Sea well versus depth. Our estimated porosity is shown in red. Right figure: Estimated overpressure and hydrostatic pressure versus depth.

Possible solution/discussion: Effect of diagenesis

Diagenesis (Figure 5) reduces the porosity, while it also makes the rock stronger. This implies that the porosity gradient changes when the diagenesis starts. At even larger depths, after the diagenesis is completed, the porosity gradient will change again. We may take this into account by the transitions

$$e^{q(z-z_w)} \rightarrow e^{q_1(z-z_1)} \cdot e^{q(z_1-z_w)} \qquad (20)$$

In the diagenesis zone ($z>z_2$), and further

$$\rightarrow e^{q_2(z-z_2)} \cdot e^{q_1(z_2-z_1)+q(z_1-z_w)} \qquad (21)$$

At the same time, r may also change in and below the diagenesis zone.

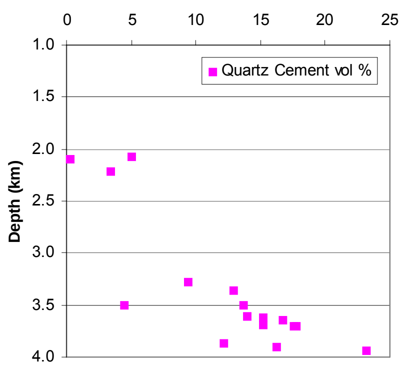

Figure 8: Example how quartz cementation increases versus depth in a reservoir unit (Garn Fm, Halten Terrace area, offshore Norway). Reworked data from Ehrenberg 1990.

The porosity may thus be written as

$$\phi = \frac{\phi_o}{r\phi_o + (1-r\phi_o)e^{q(z-z_w)}} \qquad z < z_1$$

$$\phi = \frac{\phi_1}{r_1\phi_1 + (1-r\phi_1)e^{q_1(z-z_1)}} \qquad z_1 < z < z_2 \qquad (22)$$

$$\phi = \frac{\phi_2}{r_2\phi_2 + (1-r\phi_2)e^{q_2(z-z_2)}} \qquad z_2 < z$$

where

$$\phi_1 = \frac{\phi_o}{r\phi_o + (1-r\phi_o)e^{q(z_1-z_w)}}$$

$$\phi_2 = \frac{\phi_1}{r_1\phi_1 + (1-r_1\phi_1)e^{q_1(z_2-z_1)}} \qquad (23)$$

We should expect that $q_2 < q$ since the rock has become stronger due to the cementation. It is less clear how q_1 relates to q, since q_1 combines two opposing effects.

At even larger depths, the stress state will reach the failure envelope for the rock, and the porosity will again be reduced at a larger rate.

Conclusions

The relationship between porosity and depth in a sedimentary basin has been derived on a simplified assumptions about stress-paths and rock compressibility, in combination with published relation between mean grain size and sea-floor porosity and relation between mean grain size content and clay content

The porosity-depth relationship is tested on published data from Yang & Aplin (2004) and well data from the North Sea. The results indicate that the parameter relationships generated by the neural network to a large extent agree with the theoretically based relationships generated here.

Also a good match is observed with the North Sea wells. This support the validity of the physics used to derive these relations. The relations provide a foundation for practical estimation of pore pressures and the rock mechanical parameters involved.

Acknowledgment

We would like to thank Norwegian Research Council for sponsoring Strategic Institute Program at SINTEF Petroleum Research. And Norsk Hydro ASA for allowing us to show well data from the North Sea.

References

Ehrenberg, S.N., *Relationship between diagenesis and reservoir quality in sandstones of the Garn Formation, Haltenbanken, Mid-Norwegian Continental Shelf.* American Association of Petroleum Geologists Bulletin, 1990. **74**(10): p. 1538-1558.

Plumley, W.J., *Abnormally high fluid pressure: survey of some basic principles.* AAPG Bulletin, 1980. **64**: p. 414-430.

Ramm, M. and K. Bjørlykke, *Porosity/depth trends in reservoir sandstones: assessing teh quantitiative effects of varying pore-pressure, temperature history and mineralogy, Norwegian shelf data.* Clay minerals, 1994. **29**: p. 475-490.

Shumway, G., *Sound speed and absorption studies of marine sediments by a resonance method - Part !!* Geophysics, 1960. **15**(3): p. 659-682.

Yang, Y. and A.C. Aplin, *Definition and application of mudstone porosity-effective stress relationships.* Petroleum Geoscience, 2004. **10**: p. 153-162.

SINTEF